

Statistics

Fall 2022

Lecture 27



Feb 19-8:47 AM

Testing One Population Standard Deviation:

$$H_0: \sigma = \sigma_0$$

$$H_0: \sigma \leq \sigma_0$$

$$H_0: \sigma \geq \sigma_0$$

$$H_1: \sigma \neq \sigma_0$$

$$H_1: \sigma > \sigma_0$$

$$H_1: \sigma < \sigma_0$$

TTT

RTT

LTT

Always identify the claim & Testing type.

P-value Method only:

$$CTS \quad \chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2}$$

For P-value: χ^2_{cdf} command with $df = n-1$

Proceed with testing chart.

Final conclusion must be about the claim.

Reject the claim OR FTR the claim

Dec 12-6:00 AM

Given $n=12$, $S=8$, $H_0: \sigma \leq 5$, claim is H_0 .
 $\alpha = .05$

Test the claim.
 $H_0: \sigma \leq 5$ claim
 $H_1: \sigma > 5$ RTT

P-value Method
 $P\text{-value} \leq \alpha$
 $.003 \leq .05$

H_0 invalid, H_1 valid
Invalid claim
Reject the claim

CTS $\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2} \xrightarrow{H_0}$
 $= \frac{(12-1) \cdot 8^2}{5^2} = 28.16$

$df = n-1 = 11$

$\chi^2 = 28.16$
 $P\text{-value} = \chi^2_{\alpha, df}(28.16, 11) = .003$

If we choose $\alpha = .002$ then $P\text{-value} > \alpha$
 H_0 valid, H_1 invalid
Valid claim
FTR the claim.

Dec 12-6:07 AM

Math dept. claims that standard deviation of all scores of all final exams is below 10.
 $\sigma < 10$ H_1

I took a sample of 15 final exams, and standard deviation of their scores was 7.5.
 $n=15$
 $S=7.5$

use $\alpha = .1$ to test the claim.
 $H_0: \sigma \geq 10$
 $H_1: \sigma < 10$ claim, LTT

CTS $\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2}$
 $= \frac{(15-1) \cdot 7.5^2}{10^2} = 7.875$

$df = n-1 = 14$

$P\text{-value} = \text{Area}$
 $\chi^2_{\alpha, df}(0, 7.875, 14) = .104$
 $P\text{-value} > \alpha$
 $.104 > .1$

H_0 valid, H_1 invalid \rightarrow Invalid claim \rightarrow **Reject the claim**

If we change α to .105, then $P\text{-value} \leq \alpha \Rightarrow H_0$ invalid, H_1 valid \rightarrow **Valid claim**
FTR the claim

Dec 12-6:16 AM

The College claims that Standard deviation of ages of all students is 12 yrs. $\sigma = 12$ H_0

I took a Sample of 10 students and Standard deviation of their ages was 9 yrs. $n = 10$
 $S = 9$
 \rightarrow NO $\alpha \Rightarrow$ use .05

Test the claim! $CTS \chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2}$
 $H_0: \sigma = 12$ claim
 $H_1: \sigma \neq 12$ TTT

$\chi^2_{df=9}(0, 5.063, 9)$
 $\chi^2_{df=9}(5.063, 9)$

$\chi^2_{df=9}(5.063, 9) = 5.063$
 $df = n - 1 = 9$
 $\chi^2_{df=9}(5.063, 9) = .829$

P-value $>$ α
 $.342 > .05$

H_0 Valid \neq H_1 invalid
 Valid claim \Rightarrow **FTR the claim**

If we change α to .35, then
 P-value \leq $\alpha \Rightarrow H_0$ invalid \Rightarrow Invalid claim
 $.342 \leq .35$ H_1 valid **Reject the claim**

Dec 12-6:30 AM

SG 31

Comparing two Population standard deviations
 $\sigma_1 \neq \sigma_2$

Sample 1	Sample 2
$n_1 =$	$n_2 =$
$S_1 =$	$S_2 =$

$NDf = n_1 - 1$
 $DDf = n_2 - 1$

$S_1 > S_2$

$H_0: \sigma_1 = \sigma_2$	$H_0: \sigma_1 \leq \sigma_2$	$H_0: \sigma_1 \geq \sigma_2$
$H_1: \sigma_1 \neq \sigma_2$	$H_1: \sigma_1 > \sigma_2$	$H_1: \sigma_1 < \sigma_2$
TTT	RTT	LTT

Always identify the claim & Testing Type

use P-value method only:

CTS F \Rightarrow 2-Samp F Test $CTS F = \frac{S_1^2}{S_2^2}$
 P-value P

Proceed with testing chart.

Final Conclusion must be about the claim.

use $F_{cdf}(L, U, NDf, DDf)$

Reject the claim OR FTR the claim

Dec 12-7:05 AM

Use the chart below

Sample 1	Sample 2
$n_1 = 10$	$n_2 = 12$
$S_1 = 8$	$S_2 = 5$

1) Is $S_1 > S_2$? Yes

2) $Ndf = n_1 - 1 = 10 - 1 = 9$
 $Ddf = n_2 - 1 = 12 - 1 = 11$

3) Find CTS F using formula.

$$F = \frac{S_1^2}{S_2^2} = \frac{8^2}{5^2} = 2.56$$

4) Find the area on each side of CTS F, multiply the smaller area by 2.

$P\text{-value} = 2 * \text{Smaller area} = 2(.072) = .144$

Dec 12-7:16 AM

5) Test the claim at $\alpha = .1$ that two Pop. standard deviations are the same.

$H_0: \sigma_1 = \sigma_2$ claim

$H_1: \sigma_1 \neq \sigma_2$ TTT

Sample 1	Sample 2
$n_1 = 10$	$n_2 = 12$
$S_1 = 8$	$S_2 = 5$

STAT TESTS

2-Samp F Test

Inpt: Stats

$\sigma_1 \neq \sigma_2$

CTS F = 2.56

P-value P = .144

P-value $>$ α
 $.144 > .1$

H_0 valid, H_1 Invalid \rightarrow Valid claim
 FTR the claim

If we change α to .15, then

P-value \leq α $\Rightarrow H_0$ invalid \rightarrow Invalid claim
 $.144 \leq .15$ Reject the claim

Dec 12-7:23 AM

Standard deviation of ages of 8 Female Students was 8 Yrs. $n=8$
 $S=8$

Standard deviation of ages of 10 male Students was 10 Yrs. $n=10$
 $S=10$

Group 1 must have larger standard deviation.

Males	Females
$n_1=10$	$n_2=8$
$S_1=10$	$S_2=8$

1) $Diff = n_1 - 1 = 9$
 $Diff = n_2 - 1 = 7$

2) CTS $F = \frac{S_1^2}{S_2^2} = \frac{10^2}{8^2} = 1.563$

3) Find the area on each side of CTS F, then multiply smaller area by 2.

P-Value = $2 * \text{Smaller area} = 2 * (.285) = .57 \checkmark$

Dec 12-7:32 AM

4) Test the claim that there is a difference between two pop. standard deviations.

$H_0: \sigma_1 = \sigma_2$ \rightarrow No $\alpha \Rightarrow .05$

$H_1: \sigma_1 \neq \sigma_2$ claim, TTT

Males	Females
$n_1=10$	$n_2=8$
$S_1=10$	$S_2=8$

2-Samp F Test
 Inpt: Stats

CTS $F = 1.5625$
 P-value $P = .569$

$P\text{-value} > \alpha \Rightarrow H_0 \text{ valid}$
 $.569 > .05$

H_1 Invalid \Rightarrow Invalid claim
 Reject the claim

SG 31 ✓✓✓

Dec 12-7:42 AM